Resit Mechanics & Relativity 1 – 2018-2019

Thursday January 31, 2019, 9:00 – 12:00, Aletta Jacobshal

Before you start, read the following:
There are N problems for a total of NN points
Write your name and student number on each sheet of paper
Do no separate the exam-strack and try to fit all answers on them
Spare sheets are added at the back of the stack
Make clear arguments and derivations and use correct notation
Support your arguments by clear drawings where appropriate
Write in a readable manner, illegible handwriting will not be graded

Three-vectors are printed italic + bold-face: ***p***
Four-vector are printed italic: *p*
Length of a vector is indicated by |...|: |***p***|

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Problem 1	**: ………. points out of 10**
Problem 2	**: ………. points out of 15**
Problem 3	**: ………. points out of 15**
Problem 4	**: ………. points out of 15**
Problem 5	**: ………. points out of 15**
Total	**: ………. points out of 70**

GRADE = 1 + 9(#points/70) =

Lorentz Transformation equations, with

$$\begin{aligned} t' &= \gamma(t - \beta x)\ , \qquad t = \gamma(t' + \beta x')\ , \\ x' &= \gamma(x - \beta t)\ , \qquad x = \gamma(x' + \beta t')\ , \\ y' &= y\ , \\ z' &= z\ . \end{aligned}$$

Einstein velocity transformations

$$\begin{aligned} v'_x &= \frac{v_x - \beta}{1 - \beta v_x}\ , \qquad v'_y = \frac{v_y\sqrt{1-\beta^2}}{1-\beta v_x}\ , \qquad v'_z = \frac{v_z\sqrt{1-\beta^2}}{1-\beta v_x}\ , \\ v_x &= \frac{v'_x + \beta}{1 + \beta v'_x}\ , \qquad v_y = \frac{v'_y\sqrt{1-\beta^2}}{1+\beta v'_x}\ , \qquad v_z = \frac{v'_z\sqrt{1-\beta^2}}{1+\beta v'_x}\ . \end{aligned}$$

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Problem 1 – Basics (10 points)

Indicate whether a statement is TRUE (T) or FALSE (F)
by placing an ✕ in the corresponding box: ☒.
You can make a correction by completely blacking out the wrong answer: ■
Score = #correct answers – 10 (minimum 0)

a) Albert Einstein was the first to formulate the principle of relativity T: □ F :■

b) The ratio E/m is frame-independent T: □ F :■

c) If m=0, $|\boldsymbol{p}|=E$ T: ■ F :□

d) Proper-time intervals are frame-dependent T: ■ F :□

e) Two events connected via a time-like interval cannot be causally connected T: □ F :■

f) The time-components of the four-momentum is defined as $E = \mathrm{d}\tau/\mathrm{d}t$ T: □ F :■

g) $\beta^2 + 1 = 1/\gamma^2$ T: □ F :■

h) Spacetime intervals are frame-independent T: ■ F :□

i) Because light has no mass, its momentum is zero T: □ F :■

j) Total four-momentum is conserved in collisions T: ■ F :□

k) $E^2 - m^2 = |\boldsymbol{p}|^2$ T: ■ F :□

l) The components of the four-momentum are frame dependent T: ■ F :□

m) The mass of a system of particles is equal to the sum of the particle masses T: □ F :■

n) Relativistic momentum is defined as $\boldsymbol{p} = \gamma\, m\, \boldsymbol{v}$ T: ■ F :□

o) When measured along a straight worldline, $\Delta t = \Delta\tau$ T: ■ F :□

p) Events inside each other's light cones are causally connected T: ■ F :□

q) Because the Lorentz transformation leaves the components *perpendicular* to the relative motion between two inertial frames unaffected, the corresponding velocity components are also the same in both frames T: □ F :■

r) Time dilation is frame-symmetric T: ■ F :□

s) From the perspective of one reference frame, clocks in another moving reference frame are running slower T: ■ F :□

t) Events on a line perpendicular to the x'-axis share the same time t T: □ F :■

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Problem 2 – Interstellar (15 points)

People fear Earth will be destroyed in the near future and therefore two Endurance-60 space ships are sent out to search for earth-like planets to move to. These space ships travel at a speed of 3/5*c* and leave Earth in opposite directions. The ground station on Earth will send a radio signal as a sign of life every year, so that the space explorers know their search is still useful.

a) In the spacetime interval on the next page, indicate the worldline of the spaceship traveling in the +x direction, the worldline of the first radiosignal, and the event corresponding to the reception of this radiosignal by the spaceship. Use the diagram to read the time the first signal is received in the spaceship's frame (**beware**: the grid is not precisely square). (**6 points**)

T = 2 yrs [1pt]

b) The Endurance-60 spaceship has a cylindrical shape that before launch was measured to have a length of 75 m and a diameter of 6 m. Once in flight, what will be its length and diameter as measured from Earth? Assume the spaceship's direction of motion is along its axis. (**4 points**)

D' = D = 6m [2pt]
β = 3/5, so $\gamma=1/\sqrt{(1-\beta^2)}=5/4$ [1pt] (if not used properly in next line)
L' = L/γ = 75m / 5/4 = 60m [1pt] (or [2pt] if gamma-calculation included)

c) Calculate the speed of one of the spaceships *from the perspective of the other spaceship*. Give the full calculation. (**3 points**)

Use Einstein velocity transformations with β = +3/5 and vx = -3/5 [1pt]
vx' = (vx-β)/(1-β.vx) [1pt] = (-3/5-3/5)/(1-3/5*-3/5) = -6/5 / (1+9/25) = -0.88 [1pt]
Strictly speaking, speed = |velocity|, so all components would have to be calculated. vy'=vy=0, vz'=vz=0.

d) The Endurance-X is an unmanned space-drone powered by a nuclear battery that lasts for 1 year. An earth-like planet is suspected to exist at a distance of 14 lightyears. If you assume that the drone moves at constant speed, what is the drone's minimum speed to visit the planet and send a yes/no signal about its habitability back to Earth? Give the full calculation. (**2 points**)

We need time-dilation by at least a factor 14 so that the ship travels for less than 1 year in it's one reference frame [1/2pt]: $\Delta t'=\Delta t/\gamma$ [1/2pt]. So $\gamma=1/\sqrt{(1-\beta^2)}=14$, or $\beta=1/\sqrt{(1-1/\gamma^2)}$ [1/2pt] = 0.997 [1/2pt].

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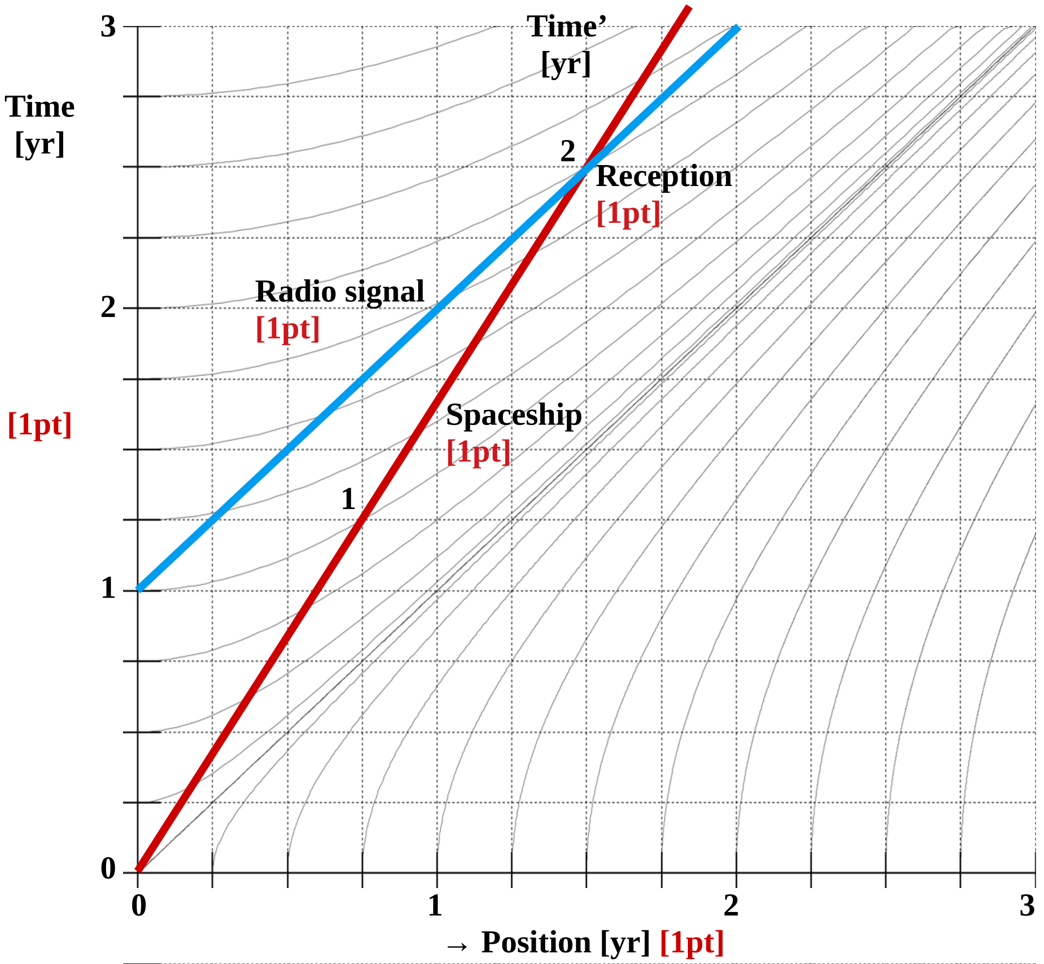
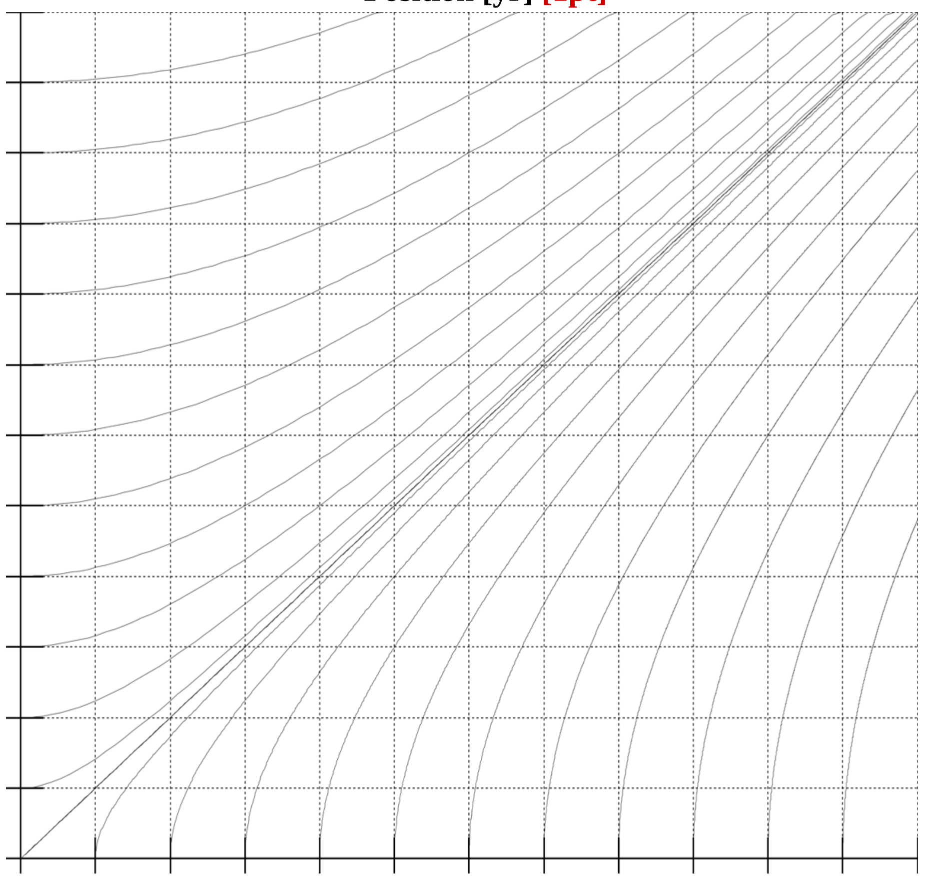


Spare diagram:



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Problem 3 – Four Momentum (15 points)

The Fermilab Tevatron, a circular accelerator located 50 miles of Chicago, can accelerate a proton p to an energy of $E_p \approx 1$ TeV = 10^{12} eV (as measured in the lab frame). The energy of a proton at rest is $m_p c^2 \approx 1$ GeV = 10^9 eV.

a) A fully accelerated proton is traveling at speed v (as measured in the lab frame) very close to the speed of light c. What is c − v, in meters per second? (**3 points**)

> **$E=\gamma m$, so $\gamma=E/m=1000$ [1pt]**
> **$\beta = \sqrt{(1-1/\gamma^2)} \approx 1-1/(2\gamma^2)$, so $(1-\beta) = \approx -1/(2\gamma^2) = ½ \times 10^{-6}$. [1pt]**
> **$(c-v) = c(1-\beta) \approx -3\times10^8$m/s / $(2\gamma^2)$ = ½ x 10^{-6} x 3×10^8m/s = 150 m/s [1pt]**

b) If a proton p with energy E_p (as measured in the lab frame) collides with a proton in a chunk of metal, at rest in the lab frame, to produce a never-before-seen particle X through the reaction p(fast) + p(at rest) → X, what is the maximum possible rest mass m_X of this particle? (**4 points**)

> **Use four-momentum conservation: $[E,p]_X = [E_1,p_1] + [E_2,p_2] = [E_p,p_p] + [m_p,0]$ [1pt]**
> **$E_X = E_p + m_p = 1001$ GeV [1pt]**
> **$P_X = p_p + 0 = \sqrt{(E_p^2 - m_p^2)} = 999.999$ GeV [1pt]**
> **$M_X = \sqrt{(E_X^2 - P_X^2)} = 45$ GeV [1pt]**

c) The anti-proton $\bar{p}$ has rest mass equal to that of the proton p, so that $m_{\bar{p}}c^2 = m_p c^2 \approx 1$ GeV. If a proton p with lab-energy E_p collides head-on with an anti-proton $\bar{p}$ of the same lab-energy E_p traveling in the *opposite* direction to produce a never-before-seen particle X through the reaction $p+\bar{p}$ → X, what is the maximum possible rest mass m_X of X? (**4 points**)

> **Use four-momentum conservation: $[E,p]_X = [E_1,p_1] + [E_2,p_2] = [E_p,p_p] + [E_p,-p_p]$ [1pt]**
> **$E_X = 2E_p = 2000$ GeV [1pt]**
> **$P_X = p_p + (-p_p) = 0$ [1pt]**
> **$M_X = \sqrt{(E_X^2 - P_X^2)} = 2000$ GeV [1pt]**

d) If the never-before-seen particle X is found to have a mass $m_X c^2 = 125$ GeV and in its rest frame decays after 10^{-12} seconds, calculate how far it will fly before decaying if it has a momentum pc = 600 GeV. (**4 points**)

> **Use time-dilation + speed: [1pt]**
> **$\gamma = E/m = 4$, so $\beta = \sqrt{(1-1/\gamma^2)} = 0.968... \approx 1$ [1pt]**
> **$L = v\cdot\Delta T_{lab} = c\cdot\beta\cdot\Delta T_{CM}\cdot\gamma \approx c\cdot\Delta T_{CM}\cdot\gamma$ [1pt]**
> **$= 3\cdot10^8$ m/s $\cdot\ 10^{-12}$ s $\cdot\ 4 = 12\cdot10^{-4}$ m = 1.2 mm [1pt]**

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Problem 4 – Fermi Problem (15 points)

What is the total combined length of all roads in Groningen?

Final answer: L[roads] = **1000 km [5pt – 1pt per factor 3]**

(Briefly) explain the variables used, including their input values (if not calculated from others):
Many different approaches possible, e.g.
D : diameter Groningen
A : area of Groningen
W : width of a road, approx. 10m
Aroad : area taken by roads
F: fraction of surface area covered by road: 3%
V: typical bicycle velocity: 20km/hr
T: time from center to Zernike: ½ hr
[5pt - ½ pt per unexplained variable]

Calculation:
Diameter of Groningen D = 2R = 2 V/T = 20km
Area of Groningen A = ¼ π D^2 = 300 km^2
Aroad = F*Agroningen = 10 km^2
Aroad = L*W → W = Aroad/W = 10 km^2 / 10 m = 10,000,000 m^2 / 10 m = 1,000 km
[5pt - ½ pt per unexplained calculation]

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Problem 5 – Dimensional Analysis (15 points)

The pressure drop $\boldsymbol{\Delta p}$ of a liquid flowing through a straight pipe of diameter $\boldsymbol{D}$ is found to depend on the length of the pipe ℓ, the velocity of the liquid $\boldsymbol{V}$, and the viscosity $\boldsymbol{\mu}$. Use dimensional analysis to find how the pressure drop per unit length (so $\boldsymbol{\Delta p}/\ell$) depends on the pipe-diameter, the velocity, and the viscosity. **Note**: pressure is measured in the unit Pascal, defined as 1Pa = 1N/m^2. The Newton (N) is the unit of force, defined from e.g. $F=m\cdot a$. Viscosity is measured in N·s/m^2.

$\Delta p/\ell = C\cdot D^{\alpha}\cdot V^{\beta}\cdot \mu^{\gamma}$ **[3pt]**

Extract dimensions (from units) [3pt]

$[\Delta p/\ell] = \mathrm{Pa\cdot m^{-1}} = \mathrm{N/m^3} = \mathrm{kg\cdot m\cdot s^{-2}\cdot m^{-3}} = \mathrm{kg\cdot m^{-2}\cdot s^{-2}} = [M\cdot L^{-2}\cdot T^{-2}]$

$[C] = -$

$[D] = [L]$

$[V] = [L\cdot T^{-1}]$

$[\mu] = \mathrm{N\cdot s\cdot m^{-2}} = \mathrm{kg\cdot m\cdot s^{-2}\cdot s\cdot m^{-2}} = \mathrm{kg\cdot m^{-1}\cdot s^{-1}} = [M\cdot L^{-1}\cdot T^{-1}]$

Equate left and right side: [3pt]

$[M\cdot L^{-2}\cdot T^{-2}] = [L]^{\alpha}\cdot[L\cdot T^{-1}]^{\beta}\cdot [M\cdot L^{-1}\cdot T^{-1}]^{\gamma} = [M]^{\gamma}\cdot[L]^{\alpha+\beta-\gamma}\cdot[T]^{-\beta-\gamma}$

Equate powers: [3pt]

[M] : $1 = \gamma$ → $\gamma = 1$

[T] : $-2 = -\beta-\gamma$ → $\beta=1$

[L] : $-2 = \alpha+\beta-\gamma$ → $\alpha=-2$

Filling in yields: [3pt]

$$\Delta p/\ell = C\cdot D^{-2}\cdot V\cdot \mu = C\cdot\mu\cdot V / D^2$$

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